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Heat transfer behavior of Reiner–Rivlin fluids in rectangular ducts

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Abstract—Numerical studies are reported for the fully developed heat transfer behavior of elastic non-Newtonian fluids in steady laminar flow through rectangular ducts. The Reiner–Rivlin formulation with finite values of the second normal stress coefficient is used to model the flow. The limiting case of zero secondary normal stress difference corresponds to a purely viscous power-law fluid. Finite difference methods are developed to obtain the heat transfer results for the H2 thermal boundary condition for different combinations of heated and adiabatic walls. The influence of the second normal stress coefficient, the Reynolds number, the Peclet Number and the aspect ratio on the heat transfer are considered. It is found that the secondary flow, which is associated with the presence of second normal stresses, results in a significant increase in the heat transfer, especially for aspect ratios of 0.5 and 1.0. The general behavior of the Nusselt number predicted for the Reiner–Rivlin fluid is found to be in good agreement with experimental results reported for viscoelastic fluids.

INTRODUCTION

The flow behavior of viscoelastic non-Newtonian fluids in circular and non-circular channels is of special engineering interest. In the limiting case of fully developed laminar flow in a circular pipe the influence of the non-Newtonian viscosity may come into play but there appears to be no influence of elastic behavior even for a viscoelastic fluid. In the case of non-circular channels the picture becomes more complex, especially for viscoelastic fluids. For constant-property, adiabatic, laminar flow of Newtonian and purely viscous non-Newtonian fluids through non-circular channels of constant cross-section there exists a main flow velocity with no secondary motions. However, in the case of viscoelastic fluids the normal stresses imposed on the orthogonal faces, which are equal for Newtonian and purely viscous non-Newtonian fluids, are unequal. This gives rise to secondary motions. This behavior has been verified analytically by Green and Rivlin [1] and Wheeler and Wissler [2].

The fluid mechanics and heat transfer behavior of aqueous polymer solutions has been studied experimentally for a number of years [3–7]. Indirect experimental evidence of secondary motions in the laminar flow of viscoelastic fluids in non-circular channels has been reported [8–14]. In one case, the measured heat transfer performance of the viscoelastic polymer solution in fully developed laminar flow was three to four times the value obtained for water [8]. A number of these investigators suggested that the high experimental values of heat transfer were caused by secondary flows resulting from the viscoelastic behavior of the non-Newtonian fluids being studied.

A comprehensive review of the laminar flow and heat transfer behavior of non-Newtonian power-law fluids in rectangular ducts has been reported by Hartnett and Kostic [15]. More recently, Gao and Hartnett analysed the flow and heat transfer behavior of a purely viscous power-law fluid [16] and the flow behavior of a Reiner–Rivlin viscoelastic fluid [17] in fully developed laminar flow through a rectangular channel by finite difference methods. In general, the effect of the secondary flow on the primary flow rate and friction factor is found to be negligible and the Reiner–Rivlin predictions agree with the power-law results.

The influence of free convection on the heat transfer behavior of viscoelastic fluids has been studied [18]. The experimental results show that for moderate concentrations of the polymer solution the enhanced heat transfer caused by secondary flow is much stronger than the free convection heat transfer. In light of this, free convection heat transfer is not considered in this analysis. Instead, fully developed laminar flow in a rectangular duct is studied to gain some insight into the high experimental value of heat transfer of viscoelastic polymer solutions in this region. A review of the literature reveals that the influence on the heat transfer of the secondary flow associated with the fully developed flow of a viscoelastic fluid in a rectangular channel has not been studied analytically. This paper deals with that problem.

MATHEMATIC FORMULATION

1. *Governing equations and boundary conditions*

Consider the steady laminar flow of an incompressible non-Newtonian fluid in a long duct of rec-

NOMENCLATURE

c^*	function of the combinations of heated and adiabatic walls
D_h	hydraulic diameter of the duct (4 cross-sectional area/perimeter) [m]
h	heat transfer coefficient [W m^{-2} K^{-1}]
k	thermal conductivity of fluid [W m^{-1} K^{-1}]
K	consistency index in power-law fluid model
L	length of the duct in Y direction [m]
$\bar{L}_h$	dimensionless heated wall length, where one heated long wall is equal to 1; one heated short wall is equal to β; two heated long walls are equal to 2; two heated short walls are equal to 2β
M	number of mesh intervals
$\bar{n}$	outer normal direction to the duct wall [m]
Nu	Nusselt number, hD_h/k
P	pressure of the fluid [Pa]
Pe^+	Peclet number, $\bar{w}D_h/\alpha$
Pr^+	Prandtl number, $(K/\rho)(\bar{w}/D_h)^{n-1}/\alpha$
q''	heat flux per unit of heating area [W m^{-2}]
Re^+	generalized Reynolds number: $\rho D_h^n \bar{w}^{2-n}/K$
T	temperature of the fluid element [K]
T_0	temperature at $z = 0$ [K]
T_m	average temperature of the fluid [K]
T_w	temperature of the wall [K]
$T_{w,m}$	average temperature of heating walls [K]
$\bar{T}$	dimensionless temperature
$\bar{T}_m$	dimensionless average temperature of the fluid
$\bar{T}_{w,m}$	dimensionless average temperature of heating walls
u, v, w	velocity component in x, y and z direction respectively [m s^{-1}]
U	dimensionless secondary flow velocity in x direction
V	dimensionless secondary flow velocity in y direction
W	dimensionless velocity in z direction
$\bar{w}$	average axial velocity [m s^{-1}]
x, y, z	rectangular Cartesian coordinates
$\bar{X}, \bar{Y}$	dimensionless coordinates in x and y axes respectively
$\Delta x, \Delta y$	dimensionless step size in x and y directions respectively.

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
α_2	second normal stress difference coefficient [N s^{-2} m^{-2}]
$\bar{\alpha}_2$	dimensionless second stress difference coefficient
β	aspect ratio
$\dot{\gamma}_{ij}$	shear rate tensor component [s^{-1}]
η	apparent viscosity of non-Newtonian fluid [N s^{-1} m^{-2}]
ρ	density of fluid [kg m^{-3}]
τ_{ij}	shear stress tensor component in j direction acting on the surface orthogonal to i direction [N m^{-2}]

tangular cross-section. The rectangular coordinate system is shown in Fig. 1(a).

The physical components of the conservation equations of mass and momentum for constant properties (ρ, k, C_p) under steady flow conditions are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xx}}{\partial x}-\frac{\partial P}{\partial x} \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}-\frac{\partial P}{\partial y} \tag{3}$$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}\right)=\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}-\frac{\partial P}{\partial z}. \tag{4}$$

A shear tensor of the Reiner–Rivlin model in a rectangular coordinate is defined as

$$\tau_{ij}=\eta\dot{\gamma}_{ij}+\alpha_2\dot{\gamma}_{ik}\dot{\gamma}_{kj} \tag{5}$$

where

$$\dot{\gamma}_{ij}=\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right). \tag{6}$$

Equations (1)–(4) were used to obtain the velocity profiles for the Reiner–Rivlin model, defined by equation (5) [17]. For the fully developed heat transfer problem, it is assumed that there is no viscous dissipation, and no energy sources within the fluid. For flow of a Reiner–Rivlin fluid involving secondary flows the energy equation can be given by the following:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{7}$$

The thermal boundary conditions to be studied involve the heating of one or more of the bounding

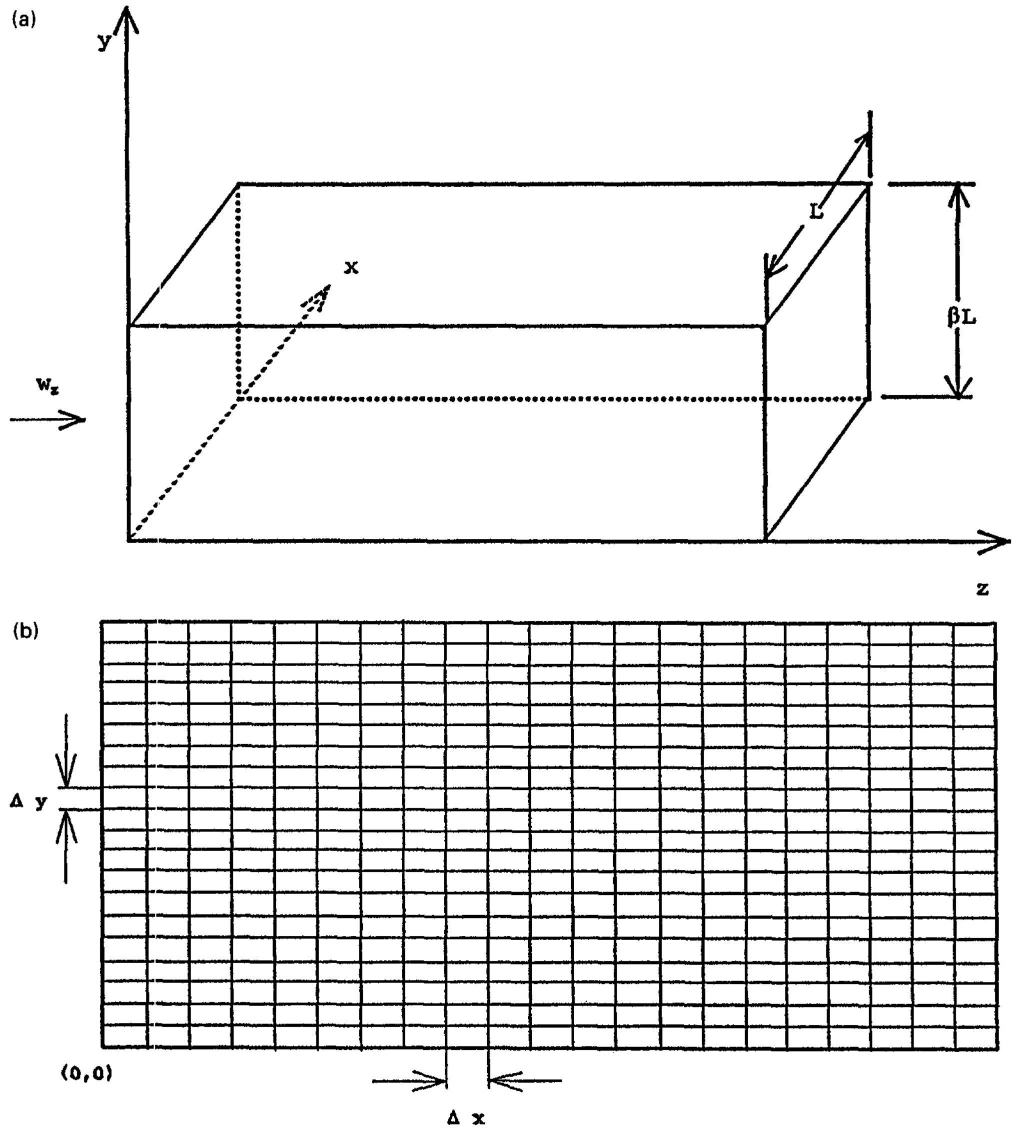

Fig. 1. (a) Rectangular duct. (b) Network over a rectangular duct for a temperature profile.

walls with the remaining walls being unheated. The heated walls are subjected to the H2 boundary condition with constant heat flux being imposed everywhere on the heated surfaces:

$$k\frac{\partial T}{\partial n} = \text{constant} \tag{8}$$

and the adiabatic wall boundary condition will be imposed on the remaining surfaces:

$$k\frac{\partial T}{\partial \bar{n}} = 0. \tag{9}$$

Eight versions of these boundary conditions involving different combinations of heated walls and adiabatic walls, as shown in Table 1, are analysed in this study.

Table 1. Different combinations of thermal boundary conditions

Symbol	Description
4	Four (all) walls heated
3L	Three walls (longer version) heated
3S	Three walls (shorter version) heated
2L	Two walls (longer version) heated
2S	Two walls (shorter version) heated
2C	Two walls (corner version) heated
1L	One wall (longer version) heated
1S	One wall (shorter version) heated
	Adiabatic (unheated) wall

Laminar flow in a rectangular duct is designated as thermally fully developed when the dimensionless fluid temperature distribution is independent of z:

Table 2. Boundary conditions for different versions and $\bar{L}_h$

Versions	$-\left.\frac{\partial\bar{T}}{\partial\bar{y}}\right\|_{\bar{y}=0}$	$-\left.\frac{\partial\bar{T}}{\partial\bar{y}}\right\|_{\bar{y}=\frac{1+\beta}{2}}$	$-\left.\frac{\partial\bar{T}}{\partial\bar{x}}\right\|_{\bar{x}=0}$	$-\left.\frac{\partial\bar{T}}{\partial\bar{x}}\right\|_{\bar{x}=\frac{1+\beta}{2\beta}}$	$\bar{L}_h$
1S	0	0	1	0	β
1L	1	0	0	0	1
2S	0	0	1	−1	2β
2L	1	−1	0	0	2
2C	1	0	1	0	$1+\beta$
3S	1	0	1	−1	$2\beta+1$
3L	1	−1	1	0	$2+\beta$
4	1	−1	1	−1	$2+2\beta$

$$\frac{\partial}{\partial z}\left(\frac{T_{w,m}-T}{T_{w,m}-T_m}\right)=0 \quad (10)$$

and

$$\frac{\partial T}{\partial z}=\text{constant}. \quad (11)$$

The non-dimensional parameters of heat transfer are defined by

$$\bar{T}=\frac{T-T_0}{q''D_h/k},\quad Nu=\frac{hD_h}{k} \quad (12)$$

where

$$q''=-k\frac{\partial T}{\partial \bar{n}}=h(T_{w,m}-T_m). \quad (13)$$

Together with the non-dimensional quantities defined above, the energy equation for the flow of a Reiner–Rivlin fluid becomes

$$\frac{\partial^2\bar{T}}{\partial\bar{x}^2}+\frac{\partial^2\bar{T}}{\partial\bar{y}^2}-Pe^+\left(U\frac{\partial\bar{T}}{\partial\bar{x}}+V\frac{\partial\bar{T}}{\partial\bar{y}}\right)=c^*W(\bar{x},\bar{y}) \quad (14)$$

with the boundary conditions

$$-\frac{\partial\bar{T}}{\partial\bar{n}}=1\ (\text{heated walls}),$$

$$-\frac{\partial\bar{T}}{\partial\bar{n}}=0\ (\text{adiabatic walls}). \quad (15)$$

Here c^* is a constant obtained from an energy balance on the flow through the duct. The magnitude of c^* is dependent on the specified combination of heated and adiabatic walls

$$c^*=\frac{2}{1+\beta}\bar{L}_h. \quad (16)$$

The details of the thermal boundary conditions and the values of $\bar{L}_h$ are shown in Table 2.

2. *Numerical approach*

The network for temperature over the rectangular duct is indicated in Fig. 1(b). For a grid size of 20 × 20 with uniform mesh intervals, ($\Delta y/\Delta x=\beta$) is applied to obtain the numerical solution.
The finite difference form of the energy equation is

$$\frac{\bar{T}_{i+1,j}-2\bar{T}_{i,j}+\bar{T}_{i-1,j}}{\Delta x^2}+\frac{\bar{T}_{i,j+1}-2\bar{T}_{i,j}+\bar{T}_{i,j-1}}{\Delta y^2}-Pe^+\times\left(U_{i,j}\frac{\bar{T}_{i+1,j}-\bar{T}_{i-1,j}}{2\Delta x}+V_{i,j}\frac{\bar{T}_{i,j+1}-\bar{T}_{i,j-1}}{2\Delta y}\right)=c^*W_{i,j} \quad (17)$$

where

$$i,j=1,2,\ldots,M-1$$

and $M=20$.

A successive over-relaxation (SOR) iteration method used to solve the sets of equations (10) is defined by

$$\bar{T}^{*k+1}_{i,j}=\frac{\Delta y^2}{2\Delta x^2+2\Delta y^2}\left[\left(1+\frac{\Delta y\,Pe^+\,U_{i,j}}{2}\right)\times\bar{T}^k_{i+1,j}+\left(1-\frac{\Delta y\,Pe^+\,U_{i,j}}{2}\right)\bar{T}^{k+1}_{i-1,j}\right]+\frac{\Delta x^2}{2\Delta x^2+2\Delta y^2}\left[\left(1+\frac{\Delta x\,Pe^+\,U_{i,j}}{2}\right)\times\bar{T}^k_{i,j+1}+\left(1-\frac{\Delta x\,Pe^+\,U_{i,j}}{2}\right)\bar{T}^{k+1}_{i,j-1}\right]-\frac{\Delta x^2\Delta y^2c^*}{2\Delta x^2+2\Delta y^2}W_{i,j} \quad (18)$$

and

$$\bar{T}^{k+1}_{i,j}=\bar{T}^k_{i,j}+\omega(\bar{T}^{*k+1}_{i,j}-\bar{T}^k_{i,j}),\quad i,j=1,2,\ldots,M-1. \quad (19)$$

The boundary conditions for the heated walls are

$$\bar{T}_{i,1} = \tfrac{1}{3}(4\bar{T}_{i,2} - \bar{T}_{i,3} - 2\Delta y), \quad \text{for} \quad \bar{y} = 0 \tag{20}$$

$$\bar{T}_{i,M+1} = \tfrac{1}{3}(4\bar{T}_{i,M} - \bar{T}_{i,M-1} - 2\Delta y), \quad \text{for} \quad \bar{y} = 1 \tag{21}$$

$$\bar{T}_{1,j} = \tfrac{1}{3}(4\bar{T}_{2,j} - \bar{T}_{3,j} - 2\Delta x), \quad \text{for} \quad \bar{x} = 0 \tag{22}$$

$$\bar{T}_{M+1,j} = \tfrac{1}{3}(4\bar{T}_{M,j} - \bar{T}_{M-1,j} - 2\Delta x), \quad \text{for} \quad \bar{x} = 1. \tag{23}$$

The boundary conditions for the adiabatic walls are

$$\bar{T}_{i,1} = \tfrac{1}{3}(4\bar{T}_{i,2} - \bar{T}_{i,3}), \quad \text{for} \quad \bar{y} = 0 \tag{24}$$

$$\bar{T}_{i,M+1} = \tfrac{1}{3}(4\bar{T}_{i,M} - \bar{T}_{i,M-1}), \quad \text{for} \quad \bar{y} = 1 \tag{25}$$

$$\bar{T}_{1,j} = \tfrac{1}{3}(4\bar{T}_{2,j} - \bar{T}_{3,j}), \quad \text{for} \quad \bar{x} = 0 \tag{26}$$

$$\bar{T}_{M+1,j} = \tfrac{1}{3}(4\bar{T}_{M,j} - \bar{T}_{M-1,j}), \quad \text{for} \quad \bar{x} = 1. \tag{27}$$

The values of $U_{i,j}$, $V_{i,j}$ and $W_{i,j}$ are the numerical results reported in [17]. A typical example of the secondary flow field for a Reiner–Rivlin fluid in fully developed laminar flow through a square duct is shown on Fig. 2. All the data symbols represent actual computational results.

RESULTS AND DISCUSSION

The numerical heat transfer results for hydrodynamically and thermally developed laminar flow for the H2 boundary condition are obtained by the successive over-relaxation iteration method. The over-relaxation coefficient ω is set to be 1.2. The number of iterations for convergence and computation time are varied from case to case. In general, as the secondary flow increases more iterations and computation time are needed. The calculated Nusselt numbers are presented as functions of the secondary normal stress coefficient, the Reynolds number Re^+, the power-law index n, the aspect ratio β and the Peclet number. The Nusselt numbers are calculated for the different combinations of heated walls using the energy equation in conjunction with the previously reported numerical values of the velocities $W_{i,j}$, $U_{i,j}$ and $V_{i,j}$ [17].

Figure 3 shows the heat transfer results in the form of the Nusselt number as a function of the second normal stress coefficient $\bar{\alpha}_2$ in a square duct at a power-law index $n = 0.7$, $Re^+ = 504$, and $Pe = 25200$ ($Pr^+ = 50$). Values of $\bar{\alpha}_2$ were selected to correspond to the estimated values of the aqueous polymer solutions used in the experimental program [18]. The other parameters are unusual numbers, such as $Re = 504$, because the convergent numerical solutions had to be normalized since the initially assumed quantity resulted in an average value of non-dimensional velocity W differing from unity. The details can be found in refs. [19] and [20]. It can be seen that the Nusselt number increases significantly as the second normal stress coefficient $\bar{\alpha}_2$ increases. For the value of $\bar{\alpha}_2 = 0.0103$, the heat transfer is about 2.5 times greater than the value corresponding to $\bar{\alpha}_2 = 0$ (power-law fluid) for the square duct which agrees with the experimental finding for viscoelastic fluids. These results demonstrate that the secondary flow, which is a result of the second normal stress difference, has a major effect on the heat transfer for non-Newtonian viscoelastic fluids in laminar flow through non-circular ducts. The stronger the secondary flow the higher the value of the heat transfer. The largest Nusselt numbers occur in the case where two opposite wall are heated for the square duct and on the 2L heating version for a 2:1 duct; the smallest Nusselt numbers occur on the corner wall heating versions.

Figure 4 shows the Nusselt number as a function of Re^+ in the square duct at a power-law index $n = 0.7$,

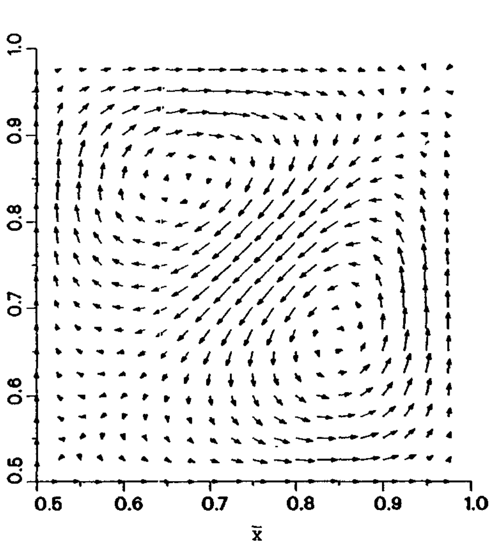

Fig. 2. Secondary flow for $\beta = 1$, $\bar{\alpha}_2 = 0.0031$, $Re^+ = 504$, $n = 0.7$.

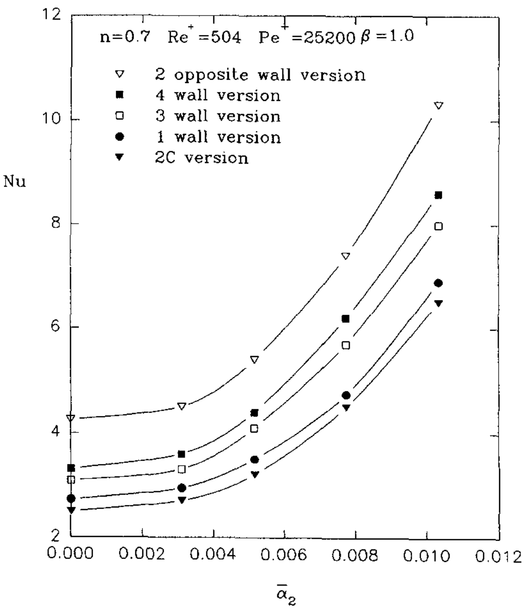

Fig. 3. Nusselt number as a function of $\bar{\alpha}_2$ in a square duct.

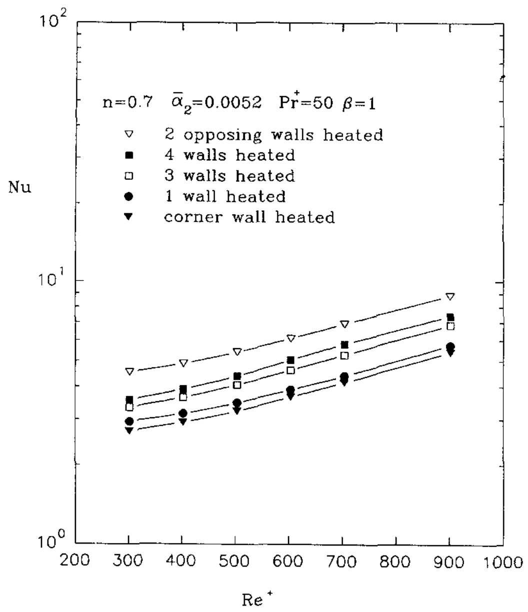

Fig. 4. Nusselt number as a function of Re^+ in a square duct.

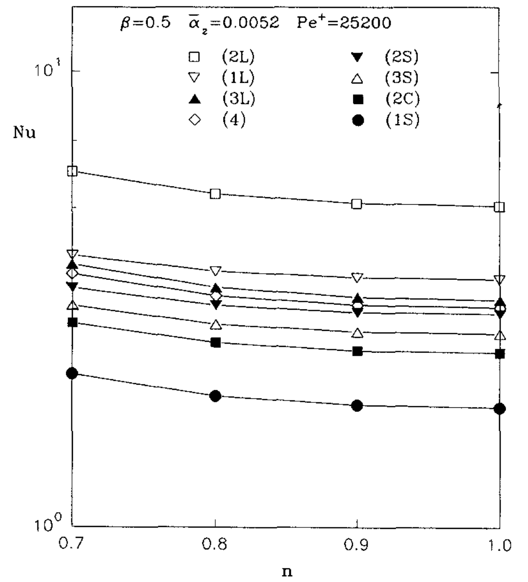

Fig. 6. Nusselt number as a function of n in a 2 : 1 duct.

$\bar{\alpha}_2 = 0.005$ and $Pr^+ = 50$. The Nusselt number corresponding to $Re^+ = 901$ is about 100% higher than Nusselt values at $Re^+ = 302$. This behavior is consistent with experimental findings [18].

Figures 5 and 6 present the Nusselt values as a function of the power-law index n for a square duct at $Re^+ = 504$, $\bar{\alpha}_2 = 0.0031$, and $Pr^+ = 50$ and for a 2 : 1 duct at $Re^+ = 504$, $\bar{\alpha}_2 = 0.0052$, and $Pr^+ = 50$. The Nusselt numbers increase as n decreases because the magnitude of secondary flow increases. In fact, at

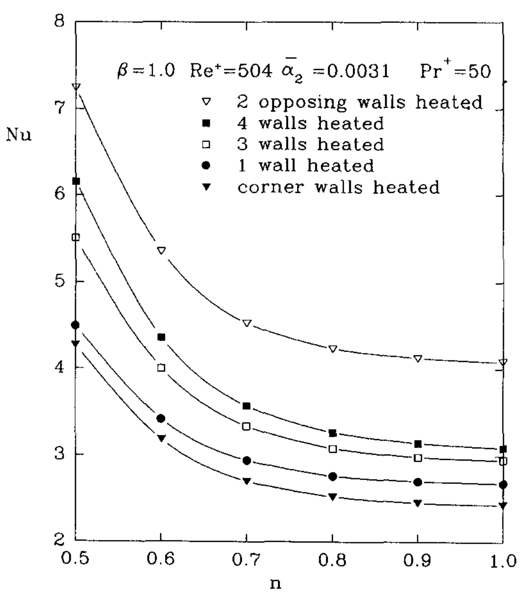

Fig. 5. Nusselt number as a function of n in a square duct.

n equal to 1, the results become equal to the values found for a power-law fluid [16] under the same heating configuration because the secondary flow vanishes for the assumed Reiner–Rivlin model at $n = 1$ [1]. The primary velocity has a negligible influence on the calculated increase in the Nusselt number relative to the value for a power-law fluid because the primary velocity for the Reiner–Rivlin fluid corresponds to the velocity for the power-law fluid for the cases studied [17].

The Nusselt numbers as a function of Peclet number are shown in Fig. 7 for the case of $\bar{\alpha}_2 = 0.0031$, $Re^+ = 504$, $n = 0.7$ and $\beta = 1$. The larger the Peclet number, which means the larger the Prandtl number Pr^+ number at a fixed Re^+, the higher the value of the Nusselt number.

Figures 8–10 present the Nusselt numbers as a function of aspect ratio for the cases of $\bar{\alpha}_2 = 0.0077$ and $\bar{\alpha}_2 = 0.0052$ with $n = 0.7$, $Re^+ = 504$ and $Pe = 25200$ for the 2L, 2C and 4 heating configurations which may be the most interesting cases inasmuch as they are associated with the maximum and minimum enhancement of heat transfer. For the 2L and 2C versions, the maximum value of Nu occurs at $\beta = 0.75$ for the cases studied. This also happens for the 1L and 3L version. For the four-wall version, there is a different tendency for the two $\bar{\alpha}_2$ values, which means that the second normal stress coefficient plays a role. At the higher value of $\bar{\alpha}_2$, the Nu values increase as the aspect ratio increases and at lower $\bar{\alpha}_2$, a maximum in the Nusselt number occurs at $\beta = 0.75$. For the 1S, 2S and 3S versions, the results show that the heat transfer increases as the aspect ratio increases due to the secondary flow. The influence of aspect ratio is complicated because the change of geometry affects

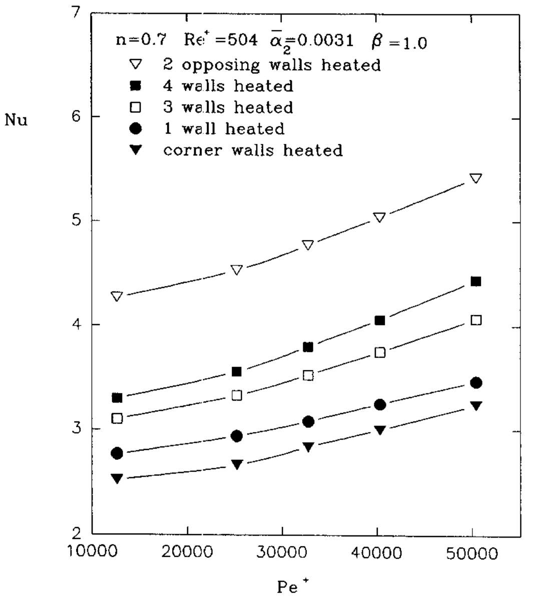

Fig. 7. Nusselt number as a function of Pe in a square duct.

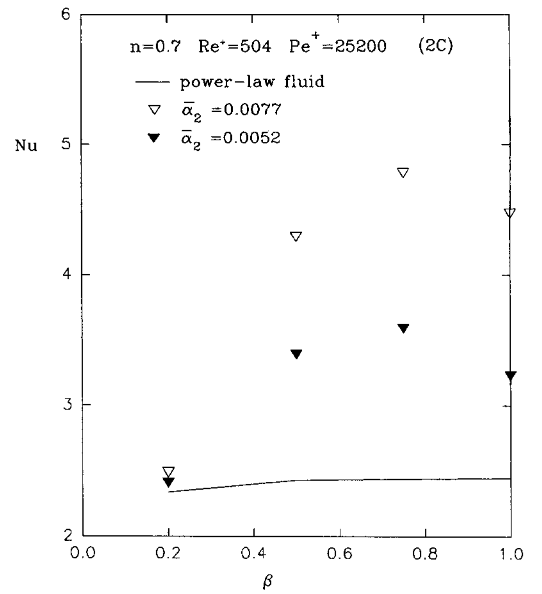

Fig. 9. Nusselt number as a function of β for the 2C version.

both the distribution and the magnitude of secondary flow. For the eight heating configurations studied the calculations reveal that the Nusselt number for a Reiner–Rivlin fluid in a rectangular duct having an aspect ratio of 0.2 does not differ appreciably from the value corresponding to a power-law fluid. The result is in agreement with experiment [19]. The values of Nusselt number for Reiner–Rivlin fluids are presented in Table 3.

From the results of this investigation, the Reiner–Rivlin model is shown to predict correct trends and to give quantitative estimates of pressure drop [17] and heat transfer behavior of viscoelastic fluid.

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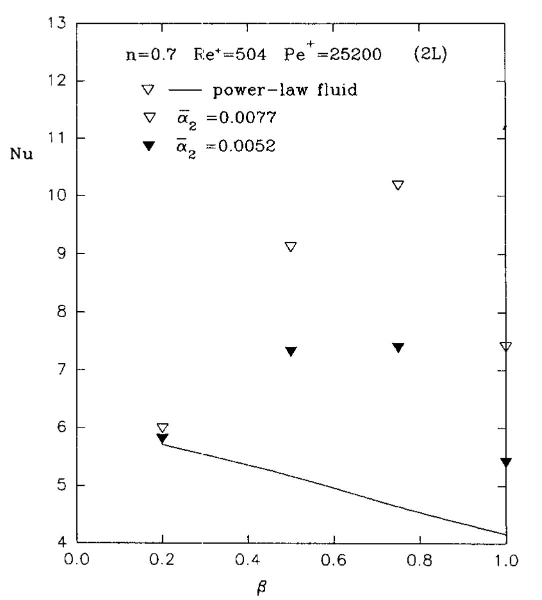

Fig. 8. Nusselt number as a function of β for the 2L version.

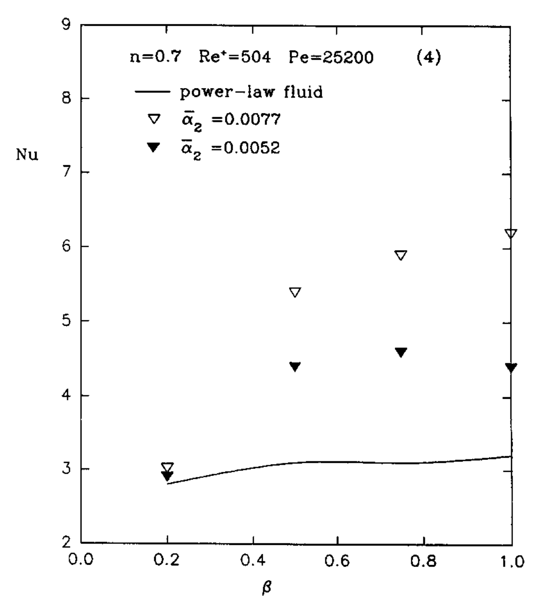

Fig. 10. Nusselt number as a function of β for the 4 version

Table 3. Heat transfer data for Reiner–Rivlin fluids in rectangular ducts ($n = 0.7$, $Pr^+ = 50$)

β	Re^+	$\bar{\alpha}_2$	1S	1L	2S	2L	2C	3S	3L	4
1.0	504	0.0103	6.93	6.93	10.34	10.34	6.48	8.04	8.04	8.56
1.0	504	0.0090	5.66	5.66	8.71	8.71	5.33	6.76	6.76	7.32
1.0	504	0.0077	4.74	4.74	7.41	7.41	4.47	5.70	5.70	6.25
1.0	503	0.0052	3.47	3.47	5.41	5.41	3.22	4.06	4.06	4.39
1.0	504	0.0031	2.94	2.94	4.53	4.53	2.70	3.33	3.33	3.56
1.0	901	0.0051	5.75	5.75	8.84	8.84	5.41	6.86	6.86	7.42
1.0	704	0.0052	4.40	4.40	6.92	6.92	4.15	5.29	5.29	5.80
1.0	604	0.0052	3.89	3.89	6.09	6.09	3.56	4.62	4.62	5.05
0.7	504	0.0077	4.05	6.57	5.25	10.18	4.78	5.07	6.31	5.87
0.5	503	0.0077	3.33	5.92	4.63	9.12	4.29	4.49	5.84	5.38
0.2	504	0.0077	1.06	4.30	1.83	5.99	2.49	2.45	3.42	3.02

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